*B1:* Premises: (P→Q), P. Conclusion: Q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (P⭢Q) | | Premise |
| 2 |  | P |  | Premise |
| 3 |  | Q |  | , |

*B2:* Premises: (P→Q), ~Q. Conclusion: ~P

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (P⭢Q) | | Premise |
| 2 |  | ~Q |  | Premise |
| 3 |  | ~P |  | , |

*B3:* Premises: ~~Q. Conclusion: Q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | ~~Q | | Premise |
| 2 |  | Q |  | , |

*B4:* Premises: ~Q, (~Q→S). Show: S.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | ~Q | | Premise |
| 2 |  | (~Q⭢S) |  | Premise |
| 3 |  | S |  | , |

*B5:* Premises: (S → ~Q), (P → S), ~~P. Show: ~Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (S⭢~Q) | | Premise |
| 2 |  | (P⭢S) |  | Premise |
| 3 |  | ~~P |  | Premise |
| 4 |  | P |  | , |
| 5 |  | S |  | , |
| 6 |  | ~Q |  | , |

*B6:* Premises: (T → P), (Q → S), (S → T), ~P. Show: ~Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (T⭢P) | | Premise |
| 2 |  | (Q⭢S) |  | Premise |
| 3 |  | (S⭢T) |  | Premise |
| 4 |  | ~P |  | Premise |
| 5 |  | ~T |  | , |
| 6 |  | ~S |  | , |
| 7 |  | ~Q |  | , |

*B7:* Premises: R, P, (P → (R → Q)). Show: Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | R | | Premise |
| 2 |  | P |  | Premise |
| 3 |  | (P⭢(R⭢Q)) |  | Premise |
| 4 |  | (R⭢Q) |  | , |
| 5 |  | Q |  | , |

*B8:* Premises: ((R → S) → Q), ~Q, (~(R → S) → V). Show: V.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | ((R⭢S) ⭢Q) | | Premise |
| 2 |  | ~Q |  | Premise |
| 3 |  | ((~(R⭢S) ⭢V) |  | Premise |
| 4 |  | ~(R⭢Q) |  | , |
| 5 |  | V |  | , |

*B9:* Premises: (P → (Q → R)), ~(Q → R). Show: ~P.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (P⭢(Q⭢R)) |  | Premise |
| 2 |  | ~(Q⭢R) |  | Premise |
| 3 |  | ~P |  | , |

*B10:* Premises: (~(Q → R) →P), ~P, Q. Show: R.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (~(Q⭢R) ⭢P) | | Premise |
| 2 |  | ~P |  | Premise |
| 3 |  | Q |  | Premise |
| 4 |  | ~~(Q⭢R) |  | , |
| 5 |  | (Q⭢R) |  | , |
| 6 |  | R |  | , |

*B11:* Premises: P,(P→R),(P→(R→Q)). Show: Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | P | | Premise |
| 2 |  | (P⭢R) |  | Premise |
| 3 |  | (P⭢(R⭢Q)) |  | Premise |
| 4 |  | R |  | , |
| 5 |  | (R⭢Q) |  | , |
| 6 |  | Q |  | , |

*C1:* Premises: P,(P→R),(P→(R→Q)). Show: Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | ~Q | | Premise |
| 2 |  | (~Q⭢S) |  | Premise |
| 3 |  |  |  | 1, 2 modus ponens |

*C2:* Premises: (S → ~Q), (P → S), ~~P. Show: ~Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (S⭢~Q) | | Premise |
| 2 |  | (P⭢S) |  | Premise |
| 3 |  | Q |  | Premise |
| 4 |  |  |  | 3, double negation |
| 5 |  |  |  | 2, 4 modus ponens |
| 6 |  |  |  | 1, 5 modus ponens |

*C3:* Premises: (T → P), (Q → S), (S → T), ~P. Show: ~Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (T⭢P) | | Premise |
| 2 |  | (Q⭢S) |  | Premise |
| 3 |  | (S⭢T) |  | Premise |
| 4 |  | ~P |  | Premise |
| 5 |  |  |  | 1, 4 modus tollens |
| 6 |  |  |  | 3, 5 modus tollens |
| 7 |  |  |  | 2, 6 modus tollens |

*C4:* Premises: R, P, (P → (R → Q)). Show: Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | R | | Premise |
| 2 |  | P |  | Premise |
| 3 |  | (P⭢(R⭢Q)) |  | Premise |
| 4 |  |  |  | 2, 5 modus ponens |
| 5 |  |  |  | 1, 4 modus tollens |

*C5:* Premises: ((R → S) → Q), ~Q, (~(R → S) → V). Show: V.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | ((R⭢S) ⭢Q) | | Premise |
| 2 |  | ~Q |  | Premise |
| 3 |  | (~(R⭢S) ⭢V) |  | Premise |
| 4 |  |  |  | 1, 2 modus tollens |
| 5 |  |  |  | 1, 4 modus ponens |

*C6:* Premises: (P → (Q → R)), ~(Q → R). Show: ~P.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (P⭢(R⭢Q)) | | Premise |
| 2 |  | ~(Q⭢R) |  | Premise |
| 3 |  |  |  | 1, 2 modus tollens |

*C7:* Premises: (~(Q → R) →P), ~P, Q. Show: R.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | (~(Q⭢R) ⭢P) | | Premise |
| 2 |  | ~P |  | Premise |
| 3 |  | Q |  | Premise |
| 4 |  |  |  | 1, 2 modus tollens |
| 5 |  |  |  | 4, double negation |
| 6 |  |  |  | 3, 5 modus ponens |

*C8:* Premises: P,(P→R),(P→(R→Q)). Show: Q.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 |  | P | | Premise |
| 2 |  | (P⭢R) |  | Premise |
| 3 |  | (P⭢(R⭢Q)) |  | Premise |
| 4 |  |  |  | 1, 2 modus ponens |
| 5 |  |  |  | 1, 3 modus ponens |
| 6 |  |  |  | 4, 5 modus ponens |