

*H1a:* Premises:  $(P \rightarrow Q) \& \sim Q$ . Conclusion:  $\sim P$

|   |                                 |   |
|---|---------------------------------|---|
| 1 | $((P \rightarrow Q) \& \sim Q)$ |   |
| 2 | $(P \rightarrow Q)$             | , |
| 3 | $\sim Q$                        | , |
| 4 | $\sim P$                        | , |

*H1b:* Premises:  $((P \rightarrow Q) \& (R \rightarrow S))$ ,  $(\sim Q \& \sim S)$ . Conclusion:  $(\sim P \& \sim R)$

|   |  |   |
|---|--|---|
| 1 | $((P \rightarrow Q) \& (R \rightarrow S))$ |   |
| 2 | $(\sim Q \& \sim S)$                       |   |
| 3 | $(R \rightarrow S)$                        | , |
| 4 | $(P \rightarrow Q)$                        | , |
| 5 | $\sim Q$                                   | , |
| 6 | $\sim S$                                   | , |
| 7 | $\sim P$                                   | , |
| 8 | $\sim R$                                   | , |
| 9 | $(\sim P \& \sim R)$                       | , |

*H1c:* Premises:  $((R \& S) \rightarrow T)$ ,  $(Q \& \sim T)$ . Conclusion:  $\sim(R \& S)$

|   |                            |   |
|---|----------------------------|---|
| 1 | $((R \& S) \rightarrow T)$ |   |
| 2 | $(Q \& \sim T)$            |   |
|   |                            |   |
| 3 | $\sim T$                   | , |
| 4 | $\sim(R \& S)$             | , |

*H1d:* Premises:  $(P \rightarrow (R \rightarrow S))$ ,  $(R \& P)$ . Conclusion:  $S$

|   |                                     |   |
|---|-------------------------------------|---|
| 1 | $(P \rightarrow (R \rightarrow S))$ |   |
| 2 | $(R \& P)$                          |   |
|   |                                     |   |
| 3 | $P$                                 | , |
| 4 | $(R \rightarrow S)$                 | , |
| 5 | $R$                                 | , |
| 6 | $S$                                 | , |

*Hint:* Premises:  $(P \rightarrow (R \rightarrow S))$ ,  $(\sim S \& P)$ . Conclusion:  $\sim R$

|   |  |                                     |   |
|---|--|-------------------------------------|---|
| 1 |  | $(P \rightarrow (R \rightarrow S))$ |   |
| 2 |  | $(\sim S \& P)$                     |   |
| 3 |  | <hr/>                               |   |
| 3 |  | P                                   | , |
| 4 |  | $(R \rightarrow S)$                 | , |
| 5 |  | $\sim S$                            | , |
| 6 |  | $\sim R$                            | , |

*H2a:* Premises:  $((P \rightarrow Q) \& \sim Q)$ . Conclusion:  $\sim P$

|   |                                 |                    |
|---|---------------------------------|--------------------|
| 1 | $((P \rightarrow Q) \& \sim Q)$ | premise            |
| 2 |                                 | 1, simplification  |
| 3 |                                 | 1, simplification  |
| 4 |                                 | 2, 3 modus tollens |

*H2b:* Premises:  $((P \rightarrow Q) \& (R \rightarrow S))$ ,  $(\sim Q \& \sim S)$ . Conclusion:  $(\sim P \& \sim R)$

|   |  |                    |
|---|--|--------------------|
| 1 | $((P \rightarrow Q) \& (R \rightarrow S))$ | premise            |
| 2 | $(\sim Q \& \sim S)$                       | premise            |
| 3 |  | 1, simplification  |
| 4 |  | 1, simplification  |
| 5 |  | 2, simplification  |
| 6 |  | 2, simplification  |
| 7 |  | 4, 5 modus tollens |
| 8 |  | 3, 6 modus tollens |
| 9 |  | 7, 8 adjunction    |

*H2c:* Premises:  $((R \& S) \rightarrow T)$ ,  $(Q \& \sim T)$ . Conclusion:  $\sim(R \& S)$

|   |                            |                    |
|---|----------------------------|--------------------|
| 1 | $((R \& S) \rightarrow T)$ | premise            |
| 2 | $(Q \& \sim T)$            | premise            |
| 3 |                            | 2, simplification  |
| 4 |                            | 1, 3 modus tollens |

*H2d:* Premises:  $(P \rightarrow (R \rightarrow S))$ ,  $(R \& P)$ . Conclusion:  $S$

|   |                                     |                   |
|---|-------------------------------------|-------------------|
| 1 | $(P \rightarrow (R \rightarrow S))$ | premise           |
| 2 | $(R \& P)$                          | premise           |
| 3 |                                     | 2, simplification |
| 4 |                                     | 1, 3 modus ponens |
| 5 |                                     | 2, simplification |
| 6 |                                     | 4, 5 modus ponens |

*H2e:* Premises:  $(P \rightarrow (R \rightarrow S))$ ,  $(\sim S \& P)$ . Conclusion:  $\sim R$

|   |                                     |                    |
|---|-------------------------------------|--------------------|
| 1 | $(P \rightarrow (R \rightarrow S))$ | premise            |
| 2 | $(\sim S \& P)$                     | premise            |
| 3 |                                     | 1, simplification  |
| 4 |                                     | 1, 3 modus ponens  |
| 5 |                                     | 2, simplification  |
| 6 |                                     | 4, 5 modus tollens |